

#The following code take as input the equation defining a hyperelliptic curve and returns as output the #corresponding tau function according to Sato's formulation

restart :

hyp := (x - 1) · (x - 1 - e) · (x - 2) · (x - 2 - e) · (x - 3) · (x - 3 - e) :

*#hyp := (x + 1 + e) * (x + 1 + 2 * e) * (x + 1 + e + e^2) * (x + 1 + 2 * e + e^2) * (x + 2 + e) * (x + 2 + 2 * e) * (x + 2 + e + e^2) * (x + 2 + 2 * e + e^2) :*

*#hyp := (x - 1) * (x - 1 - e) * (x - 2) * (x - 2 - e) * (x - 3) * (x - 3 - e) · (x - 4) · (x - 4 - e) · (x - 5) · (x - 5 - e) :*

d := degree(hyp, x) :

x := 1/z :

*y := (expand(factor(hyp * z^d)))^(1/2) :*

#Compute the series expansion of y

M := 4: #order of e to appear

N := 20 :

S := mtaylor(mtaylor(y, e, M), z, N + d/2) :

n := d/2 :

m := n - 1 :

#define the array α of the coefficients α_i

$\alpha := \text{Array}(0..N + n + m) :$

$\alpha[0] := 1 :$

for i to N + n do

$\alpha[i] := \text{factor}(\text{coeff}(S, z, i))$

end do:

#define the array of the functions $g_n, \dots, g_{(N+n+m)}$

$g := \text{Array}(1..N + n + m) :$

for i to N + n do

for j to i + n do

$g[i + m] := \alpha[j - 1] \cdot x^{i + n - j} + g[i + n - 1]$

end do:

end do:

#define the array of the functions $f_n, \dots, f_{(N+n+m)}$

$f := \text{Array}(1..N + n + m) :$

for i to N + n do

$f[i + m] := \text{expand}\left(\text{factor}\left(\frac{1}{2} \cdot (x^{i + m} \cdot y + g[i + m]) \cdot z^{i + m}\right)\right)$

end do:

#define the array with the series expansions of the functions $1, f_n, \dots, f_{(N+n+3)}$

Cmatrix := Array(1..N + n + 1) :

Cmatrix[1] := taylor(z^m, z, N) :

for i to N + n do

$Cmatrix[i + 1] := \text{mtaylor}(\text{mtaylor}(f[i + m], e, M), z, i + 1 + N + m)$

end do:

#compute the matrix corresponding to the frame{1, f_n, ..., f_(N+n+m)}

$\xi := (i, j) \rightarrow \text{factor}(\text{coeff}(\text{Cmatrix}[N+n+2-j], z, i-j)) :$

$A := \text{Matrix}(2 \cdot (N+n), N+n+1, \xi);$

$\text{numRows} := 2 \cdot (N+n) + 1 :$

=

$$A := \begin{bmatrix} 1 & 0 & 0 & 0 \\ -6 - \frac{3e}{2} & 1 & 0 & 0 \\ 11 + 6e + \frac{3}{8}e^2 & -6 - \frac{3e}{2} & 1 & 0 \\ \frac{(4+e)(e^2-16e-24)}{16} & 11 + 6e + \frac{3}{8}e^2 & -6 - \frac{3e}{2} & 1 \\ -\frac{3e^2}{8} & \frac{(4+e)(e^2-16e-24)}{16} & 11 + 6e + \frac{3}{8}e^2 & -6 - \frac{3e}{2} \\ -\frac{3e^2(4+e)}{16} & -\frac{3e^2}{8} & \frac{(4+e)(e^2-16e-24)}{16} & 11 + 6e + \frac{3}{8}e^2 \\ -\frac{e^2(8+3e)}{4} & -\frac{3e^2(4+e)}{16} & -\frac{3e^2}{8} & \frac{(4+e)(e^2-16e-24)}{16} \\ -3e^2(e+2) & -\frac{e^2(8+3e)}{4} & -\frac{3e^2(4+e)}{16} & -\frac{3e}{8} \\ -\frac{e^2(37+24e)}{2} & -3e^2(e+2) & -\frac{e^2(8+3e)}{4} & -\frac{3e^2(4+e)}{16} \\ -\frac{e^2(228+185e)}{4} & -\frac{e^2(37+24e)}{2} & -3e^2(e+2) & -\frac{e^2(8+3e)}{4} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$\text{partLength} := 8 :$

$p = \text{Array}(1..\text{partLength}) :$

for i **to** partLength **do**

$p[i] := 0$

end do:

$n1 := 0 : n2 := 0 : n3 := 0 :$

for i **to** partLength **do**

for $n1$ **from** 0 **to** i **do**

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for n2 from 0 to i do
  for n3 from 0 to i do
    if n1 + 2·n2 + 3·n3 = i then
      
$$p[i] := p[i] + \frac{(X^{n1} \cdot Y^{n2} \cdot T^{n3})}{n1! \cdot n2! \cdot n3!} :$$

    end if:
  end do:
end do:
end do:
end do:

```

#make one only large vector with all the partitions together
with (combinat, partition) : with (ListTools) :

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partitions := FlattenOnce([seq(partition(j), j = 1 .. partLength)]) :
h := numelems(partitions) :

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#make the corresponding vector of Maya diagrams

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Maya = Array(1 .. h) :
for i to h do
  r := numelems(partitions[i]) :
  Maya[i] := Array(1 .. r) :
  for j to r do
    Maya[i][j] := partitions[i][r - j + 1] - j + N + n + 2
  end do:
end do:
eqns := {e4 = 0} :
τ := 0 :

```

#compute the coordinates, the schur polynomial for the non zero coordinates and the tau function

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Coordinates = Array(1 .. h) :
with (LinearAlgebra) : with (ArrayTools) : with (linalg) :
for i to h do

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  r := numelems(Maya[i]) :
  increasingMaya := convert(FlipDimension(Maya[i], 2), list) :
  if Maya[i][1] < numRows then
    #this condition is needed for Maya diagrams corresponding to rows we did not compute
    Coordinates[i] := simplify(Determinant(SubMatrix(A, increasingMaya, [N + n + 1 - r
    + 1 .. N + n + 1])), eqns) :
  else
    Coordinates[i] := 0 :
  end if:
if Coordinates[i] ≠ 0 then
  l := Array(1 .. r) :
  v := Vector(1 .. r) :
  for k to r do
    l[k] := partitions[i][k] + (k - 1) :
    v[k] := p[l[k]] :
  end do:

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$Wr := \text{wronskian}(v, X) :$

$\text{schur} := \det(Wr) :$

$\tau := \tau + \text{Coordinates}[i] \cdot \text{schur} :$

end if:

end do:

$\tau;$
=

$$\begin{aligned} & \left(\frac{1}{2} X Y^2 T + \frac{1}{2} Y T^2 + \frac{1}{6} X^3 Y T + \frac{1}{6} X^2 Y^3 + \frac{1}{12} X^4 Y^2 + \frac{1}{4} X^2 T^2 + \frac{1}{120} X^5 T + \frac{1}{72} X^6 Y \right. \\ & \quad \left. + \frac{1}{1440} X^8 - \frac{1}{6} Y^4 \right) \left(\frac{3}{8} e^3 + e^2 \right) + \left(-\frac{1}{2} X Y^2 T + \frac{1}{24} X^4 Y^2 + \frac{1}{6} X^3 Y T + \frac{1}{4} X^2 T^2 \right. \\ & \quad \left. - \frac{1}{24} X^5 T + \frac{1}{72} X^6 Y + \frac{1}{576} X^8 - \frac{1}{12} Y^4 - \frac{1}{6} X^2 Y^3 + \frac{1}{2} Y T^2 \right) \left(-\frac{3}{2} e^3 - \frac{13}{4} e^2 \right) \\ & \quad + \left(\frac{1}{24} X^4 Y^2 + X Y^2 T - \frac{1}{20} X^5 T + \frac{1}{960} X^8 - \frac{1}{4} Y^4 \right) \left(\frac{9}{4} e^3 + \frac{69}{16} e^2 \right) + \left(\frac{1}{3} X^3 Y T \right. \\ & \quad \left. - \frac{1}{180} X^6 Y - \frac{1}{30} X^5 T + \frac{1}{720} X^8 - \frac{1}{3} X^2 Y^3 - \frac{1}{2} Y T^2 + \frac{1}{3} Y^4 - \frac{1}{4} X^2 T^2 \right) \left(\frac{15}{4} e^3 \right. \\ & \quad \left. + \frac{127}{16} e^2 \right) + \left(\frac{1}{4} Y^4 + \frac{1}{24} X^4 Y^2 - \frac{1}{2} X Y^2 T - \frac{1}{180} X^6 Y - \frac{1}{6} X^3 Y T + Y T^2 \right. \\ & \quad \left. + \frac{1}{2880} X^8 - \frac{1}{120} X^5 T + \frac{1}{2} X^2 T^2 \right) \left(575 + \frac{333}{2} e^3 + \frac{1085}{2} e^2 + 888 e \right) + \left(-\frac{1}{72} X^6 Y \right. \\ & \quad \left. - \frac{1}{6} X^3 Y T + \frac{1}{12} X^4 Y^2 + \frac{1}{1440} X^8 + \frac{1}{120} X^5 T + \frac{1}{4} X^2 T^2 + \frac{1}{2} X Y^2 T - \frac{1}{6} Y^4 \right. \\ & \quad \left. - \frac{1}{6} X^2 Y^3 - \frac{1}{2} Y T^2 \right) \left(1729 + \frac{2115}{4} e^3 + 1599 e^2 + 2592 e \right) + \left(\frac{1}{2016} X^8 - \frac{1}{72} X^6 Y \right. \\ & \quad \left. + \frac{1}{24} X^5 T + \frac{1}{12} X^4 Y^2 - \frac{1}{6} X^3 Y T - \frac{1}{6} X^2 Y^3 - \frac{1}{4} X^2 T^2 + \frac{1}{2} X Y^2 T + \frac{1}{6} Y^4 \right. \\ & \quad \left. - \frac{1}{2} Y T^2 \right) \left(2771 + \frac{3717}{4} e^3 + \frac{19875}{8} e^2 + 3906 e \right) + \left(\frac{1}{5760} X^8 - \frac{1}{144} X^6 Y + \frac{1}{30} X^5 T \right. \\ & \quad \left. + \frac{1}{16} X^4 Y^2 - \frac{1}{3} X^3 Y T - \frac{1}{12} X^2 Y^3 + \frac{1}{4} X^2 T^2 - \frac{1}{24} Y^4 + \frac{1}{2} Y T^2 \right) \left(3025 + 1113 e^3 \right. \\ & \quad \left. + \frac{21243}{8} e^2 + 3864 e \right) \\ & \quad + \frac{1}{32} \left(3 \left(-\frac{1}{2} Y^2 T + \frac{1}{6} X^3 Y^2 + \frac{1}{2} X^2 Y T + X T^2 - \frac{1}{24} X^4 T + \frac{1}{30} X^5 Y \right. \right. \\ & \quad \left. \left. + \frac{1}{360} X^7 \right) e^2 (4 + e) \right) \\ & \quad - \frac{1}{8} \left(3 \left(\frac{1}{12} X^3 Y^2 + \frac{1}{2} X^2 Y T + \frac{1}{120} X^5 Y - \frac{1}{8} X^4 T + \frac{1}{240} X^7 - \frac{1}{2} X Y^3 \right. \right. \\ & \quad \left. \left. + \frac{1}{2} Y^2 T \right) e^2 (4 + e) \right) + \left(-\frac{1}{30} X^5 Y - \frac{1}{2} X^2 Y T + \frac{1}{6} X^3 Y^2 + \frac{1}{360} X^7 - \frac{1}{24} X^4 T \right. \\ & \quad \left. + X T^2 - \frac{1}{2} Y^2 T \right) \left(-444 - \frac{1013}{16} e^3 - \frac{1113}{4} e^2 - 571 e \right) + \left(\frac{1}{6} X^3 Y^2 + \frac{1}{360} X^7 \right. \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{20} X^5 Y - \frac{1}{3} XY^3 + Y^2 T + \frac{1}{12} X^4 T - \frac{1}{2} XT^2 \left(-840 - \frac{283}{2} e^3 - 516 e^2 - 1018 e \right) \\
& + \left(\frac{1}{840} X^7 - \frac{1}{30} X^5 Y + \frac{1}{8} X^4 T + \frac{1}{6} X^3 Y^2 - \frac{1}{2} X^2 YT - \frac{1}{2} Y^2 T \right) \left(-966 - \frac{3073}{16} e^3 \right. \\
& \left. - \frac{2373}{4} e^2 - \frac{2107}{2} e \right) \\
& + \frac{3 \left(\frac{1}{4} X^2 Y^2 + XY T + \frac{1}{24} X^4 Y + T^2 - \frac{1}{6} X^3 T + \frac{1}{144} X^6 - \frac{1}{2} Y^3 \right) e^2}{16} + \left(\frac{1}{2} Y^3 \right. \\
& \left. - \frac{1}{24} X^4 Y - XY T + \frac{1}{4} X^2 Y^2 + \frac{1}{144} X^6 - \frac{1}{6} X^3 T + T^2 \right) \left(85 + \frac{15}{4} e^3 + \frac{63}{2} e^2 + 90 e \right) \\
& + \left(\frac{1}{4} X^2 Y^2 + \frac{1}{80} X^6 - \frac{1}{8} X^4 Y - \frac{1}{2} Y^3 \right) \left(239 + 15 e^3 + \frac{711}{8} e^2 + 240 e \right) + \left(\frac{1}{144} X^6 \right. \\
& \left. - \frac{1}{8} X^4 Y + \frac{1}{3} X^3 T + \frac{1}{6} Y^3 + \frac{1}{4} X^2 Y^2 - \frac{1}{2} T^2 \right) \left(301 + \frac{105}{4} e^3 + \frac{939}{8} e^2 + 270 e \right) \\
& + \left(\frac{1}{2} XY^2 + \frac{1}{24} X^5 - \frac{1}{2} X^2 T - \frac{1}{6} X^3 Y - YT \right) \left(-60 - 47 e - \frac{21}{2} e^2 - \frac{5}{8} e^3 \right) \\
& + \left(\frac{1}{30} X^5 - \frac{1}{3} X^3 Y + YT + \frac{1}{2} X^2 T \right) \left(-90 - \frac{125}{2} e - \frac{75}{4} e^2 - \frac{35}{16} e^3 \right) + \left(Y^2 \right. \\
& \left. + \frac{1}{12} X^4 - XT \right) \left(11 + 6 e + \frac{3}{8} e^2 \right) + \left(\frac{1}{8} X^4 - \frac{1}{2} Y^2 - \frac{1}{2} X^2 Y \right) \left(25 + 12 e + \frac{15}{8} e^2 \right) \\
& + \left(\frac{X^3}{3} - T \right) \left(-6 - \frac{3e}{2} \right) + Y + \frac{X^2}{2}
\end{aligned}$$

$$t := (X, Y, T) \rightarrow \tau :$$

#Hirota bilinear form (c,d=0)

$$\tau_x := \text{diff}(t(X, Y, T), X) :$$

$$\tau_{xx} := \text{diff}(\tau_x, X) :$$

$$\tau_{xxx} := \text{diff}(\tau_{xx}, X) :$$

$$\tau_{xxxx} := \text{diff}(\tau_{xxx}, X) :$$

$$\tau_t := \text{diff}(t(X, Y, T), T) :$$

$$\tau_{xt} := \text{diff}(\tau_x, T) :$$

$$\tau_y := \text{diff}(t(X, Y, T), Y) :$$

$$\tau_{yy} := \text{diff}(\tau_y, Y) :$$

$$\text{Hirota} := \text{simplify}\left((\tau_{xxxx} \cdot \tau - 4 \cdot \tau_{xxx} \cdot \tau_x + 3 \tau_{xx}^2) + 4 \cdot (\tau_x \cdot \tau_t - \tau \cdot \tau_{xt}) + 3 \cdot (\tau \cdot \tau_{yy} - \tau_y^2), \text{eqns} \right);$$

$$= \frac{1}{9676800} (53180287731 e^3 + 62180312507 e^2 + 46032402864 e + 16418043360) X^{12}$$

$$+ \frac{1}{9676800} (-77163444234 e^3 - 99482565672 e^2 - 80581073088 e - 31270245120) X^{11}$$

$$+ \frac{1}{9676800} ((-497616372972 Y + 56353321800) e^3 + (-575651421072 Y$$

$$+ 80648566482) e^2 + (-428084436672 Y + 72129873600) e - 156321532800 Y$$

$$\begin{aligned}
& + 30860300160) X^{10} + \frac{1}{9676800} ((-2947858256880 T + 465374164440 Y \\
& - 17567149560) e^3 + (-3481930339080 T + 590285242080 Y - 27504684000) e^2 + (- \\
& -2582468530560 T + 480231649920 Y - 27305630400) e - 910151043840 T \\
& + 191998886400 Y - 13322545920) X^9 + \frac{1}{9676800} ((2724929893260 Y^2 + 3206793777570 T \\
& - 248896132560 Y - 2950598880) e^3 + (3197656355220 Y^2 + 4182645512520 T \\
& - 350065468260 Y - 6727477680) e^2 + (2383906092480 Y^2 + 3396337983360 T \\
& - 314400251520 Y - 8007465600) e + 857150703360 Y^2 + 1301466216960 T \\
& - 138493491840 Y - 3725398080) X^8 + \frac{1}{9676800} ((-986989778640 Y^2 + (- \\
& -5322428611200 T + 117764808480) Y - 1120691039040 T + 2755198080) e^3 + (- \\
& -1311041574720 Y^2 + (-6190198225920 T + 190153802880) Y - 1633576053600 T \\
& + 6592043520) e^2 + (-1076102208000 Y^2 + (-4758550778880 T + 190960289280) Y \\
& - 1463737236480 T + 8883417600) e - 410288302080 Y^2 + (-1835073792000 T \\
& + 91423365120) Y - 612169021440 T + 5135616000) X^7 + \frac{1}{9676800} ((14772789511200 Y^3 \\
& - 1415300402880 Y^2 + (1675466205840 T - 50175780480) Y + 73298837183040 T^2 \\
& - 321575985360 T) e^3 + (17060419561920 Y^3 - 1997500009680 Y^2 + (2067938994240 T \\
& - 96887992320) Y + 86028145338240 T^2 - 530832294720 T) e^2 + (12756782062080 Y^3 \\
& - 1785620712960 Y^2 + (1828388782080 T - 110920320000) Y + 64328998970880 T^2 \\
& - 538586012160 T) e + 4723982592000 Y^3 - 777963755520 Y^2 + (864757555200 T \\
& - 57644536320) Y + 23264245509120 T^2 - 255577835520 T) X^6 + \frac{1}{9676800} ((\\
& -16348763249280 Y^3 + (-186172678419840 T + 1547421926400) Y^2 + (6470387360640 T \\
& + 7607355840) Y - 39351779691840 T^2 + 226134840960 T) e^3 + (-20653944675840 Y^3 + (- \\
& -217383696734400 T + 2489147781120) Y^2 + (9363878959680 T + 18681788160) Y \\
& - 51021650131200 T^2 + 430600302720 T) e^2 + (-16852226872320 Y^3 + (- \\
& -162327095884800 T + 2493077045760) Y^2 + (8386191636480 T + 25077427200) Y \\
& - 41665700160000 T^2 + 493720012800 T) e - 6823711641600 Y^3 + (-58937456148480 T \\
& + 1194165504000) Y^2 + (3530806917120 T + 14224896000) Y - 16287325931520 T^2 \\
& + 260517841920 T) X^5 + \frac{1}{9676800} ((29762359880400 Y^4 + 4310464636800 Y^3 \\
& + (109241417048400 T - 294230361600) Y^2 + (518857706505600 T^2 - 5000496530400 T) Y \\
& - 966236040000 T^2 + 24456146400 T) e^3 + (36376344925200 Y^4 + 5776411888800 Y^3 \\
& + (140342057352000 T - 519900595200) Y^2 + (604971577224000 T^2 - 8055328176000 T) Y \\
& - 1355169916800 T^2 + 55420848000 T) e^2 + (27099598982400 Y^4 + 5152997606400 Y^3 \\
& + (114444392832000 T - 581928883200) Y^2 + (452813232614400 T^2 - 8104784486400 T) Y \\
& - 1289341670400 T^2 + 73703347200 T) e + 9125352230400 Y^4 + 2409299020800 Y^3 \\
& + (45155219558400 T - 323604288000) Y^2 + (165784631040000 T^2 - 3888447897600 T) Y
\end{aligned}$$

$$\begin{aligned}
& - 613209139200 T^2 + 43835904000 T) X^4 + \frac{1}{9676800} \left((-824960656800 Y^4 + (\right. \\
& -448422512832000 T + 113382864000) Y^3 + (-1811159481600 T - 141176044800) Y^2 + (\\
& -211593344860800 T^2 + 401294476800 T) Y - 431513479353600 T^3 + 1181216131200 T^2) e^3 \\
& + (-2370656937600 Y^4 + (-529716309696000 T + 375588057600) Y^3 + (-2462621616000 T \\
& - 330873984000) Y^2 + (-272314695744000 T^2 + 685436774400 T) Y - 512283569817600 T^3 \\
& + 2127746880000 T^2) e^2 + (-1933593984000 Y^4 + (-395571385036800 T \\
& + 430175692800) Y^3 + (-2004316876800 T - 445277952000) Y^2 + (-222270676992000 T^2 \\
& + 771163545600 T) Y - 380953514188800 T^3 + 2167206451200 T^2) e - 146526105600 Y^4 \\
& + (-141609000960000 T + 124211404800) Y^3 + (-766557388800 T - 260692992000) Y^2 \\
& + (-87787929600000 T^2 + 444300595200 T) Y - 133715648102400 T^2 \left(T - \frac{385}{54834} \right) \Big) X^3 \\
& + \frac{1}{9676800} \left((244039589640000 Y^5 - 24001712380800 Y^4 + (117745511889600 T \\
& + 1034012044800) Y^3 + (1367631313977600 T^2 - 5934135816000 T) Y^2 \right. \\
& + (15785595705600 T^2 - 90932284800 T) Y + 91209418977600 T^3 + 1371893241600 T^2) e^3 \\
& + (285401026713600 Y^5 - 34050758767200 Y^4 + (155718996652800 T + 1906423948800) Y^3 \\
& + (1616849183980800 T^2 - 9731581574400 T) Y^2 + (22669389388800 T^2 \\
& - 223802611200 T) Y + 123385684665600 T^3 + 2398388227200 T^2) e^2 \\
& + (213236576179200 Y^5 - 30687789081600 Y^4 + (126528456499200 T + 2163519590400) Y^3 \\
& + (1205348043110400 T^2 - 9822232627200 T) Y^2 + (20495520460800 T^2 \\
& - 300929126400 T) Y + 99524615500800 T^3 + 2704181760000 T^2) e + 77533182720000 Y^5 \\
& - 13472689305600 Y^4 + (47767781376000 T + 1165677004800) Y^3 + (429045158707200 T^2 \\
& - 4662862848000 T) Y^2 + (8849569075200 T^2 - 170698752000 T) Y + 35406578995200 T^3 \\
& + 1526157158400 T^2) X^2 + \frac{1}{9676800} \left((-41350905993600 Y^5 + (-1100892777465600 T \\
& + 4203206985600) Y^4 + (25580057126400 T + 29234822400) Y^3 + (-364572457804800 T^2 \\
& + 2576397196800 T) Y^2 + (-1159567443417600 T^3 + 825138316800 T^2) Y \right. \\
& - 21669727564800 T^3 + 288493228800 T^2) e^3 + (-53477545766400 Y^5 + (\\
& -1298923285161600 T + 6844562726400) Y^4 + (37288113004800 T + 73200153600) Y^3 + (\\
& -476260665753600 T^2 + 4878072460800 T) Y^2 + (-1369514018880000 T^3
\end{aligned}$$

$$\begin{aligned}
& + 1390933555200 T^2) Y - 31117547865600 T^3 + 658714291200 T^2) e^2 + (\\
& -43486329600000 Y^5 + (-964904916326400 T + 6873706828800) Y^4 + (33370212556800 T \\
& + 100309708800) Y^3 + (-388772649216000 T^2 + 5583242649600 T) Y^2 + (\\
& -1018816546406400 T^3 + 1424231424000 T^2) Y - 28426567680000 T^3 + 884440166400 T^2) e \\
& - 16968268800000 Y^5 + (-342103774924800 T + 3260404224000) Y^4 + (13937572454400 T \\
& + 56899584000) Y^3 + (-150170659430400 T^2 + 2956165632000 T) Y^2 \\
& - 361393643520000 \left(T - \frac{23}{12350} \right) T^2 Y - 12465886003200 \left(T - \frac{755}{17892} \right) T^2) X \\
& + \frac{1}{9676800} (396345125332800 Y^6 - 30148255468800 Y^5 + (241920947872800 T \\
& + 732618432000) Y^4 + (1943324739417600 T^2 - 23202575625600 T) Y^3 + (\\
& -3723965280000 T^2 + 290469110400 T) Y^2 + (289157253840000 T^3 - 4474636992000 T^2) Y \\
& + 221478985324800 T^4 + 820009008000 T^3) e^3 + \frac{1}{9676800} (466542495921600 Y^6 \\
& - 43843222382400 Y^5 + (312501759696000 T + 1385527852800) Y^4 \\
& + (2258995262227200 T^2 - 37940562777600 T) Y^3 + (-3436270387200 T^2 \\
& + 661392345600 T) Y^2 + (368543417356800 T^3 - 8353058515200 T^2) Y \\
& + 258182843990400 T^4 + 1475738611200 T^3) e^2 + \frac{1}{9676800} (347172061824000 Y^6 \\
& - 39442298112000 Y^5 + (254653652736000 T + 1586598451200) Y^4 \\
& + (1694134542643200 T^2 - 38198355148800 T) Y^3 + (-3532941619200 T^2 \\
& + 884440166400 T) Y^2 + (303220060416000 T^3 - 9531609292800 T^2) Y \\
& + 192682138521600 T^4 + 1455090739200 T^3) e + \frac{38427136}{3} Y^6 - 1725849 Y^5 \\
& + \frac{1}{9676800} (99658227916800 T + 840463948800) Y^4 + \frac{1}{9676800} (624656793600000 T^2 \\
& - 18057373286400 T) Y^3 + \frac{1}{9676800} (-2682176716800 T^2 + 526030848000 T) Y^2 \\
& + \frac{1}{9676800} (123134764032000 T^3 - 5100699340800 T^2) Y + 7282548 \left(T + \frac{1}{117} \right) T^3
\end{aligned}$$